RC Circuits

Objective

To calculate the capacitance of a capacitor by measuring the time-constant τ for charging and discharging a capacitor and compare with the expected measured value using a capacitance meter.

Equipment

- Circuit board
- Wire jumpers
- Leads
- 1 resistor $\approx 1\text{k}\Omega$
- 1 capacitor
- Capacitance meter
- Handheld DMM
- 3 BNC cables
- Oscilloscope
- Function generator

Theory

When we apply a voltage differential to two conducting plates separated by a non-conducting material, we observe that the plates will accumulate charge. The relationship between the amount of charge and the voltage differential is given by:

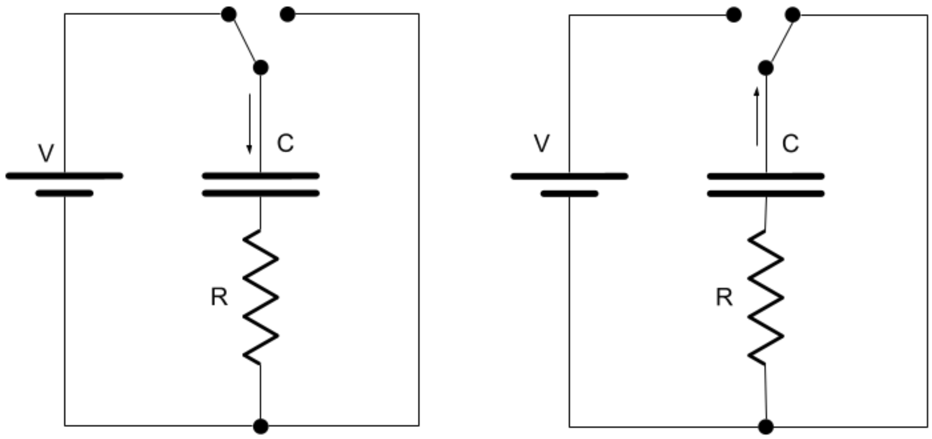
$$C = \frac{Q}{V_C} \;\rightarrow\; Q = CV_C \tag{1}$$

If we take the derivative of this with respect to time, we get:

$$\frac{\partial Q}{\partial t} = C\frac{\partial V_C}{\partial t} \;\rightarrow\; I(t) = C\frac{\partial V_C}{\partial t} \tag{2}$$

where we denote V_C as the voltage across the capacitor.

Considering the circuit below:



Charging the capacitor　　Discharging the capacitor

Charging Phase

In the charging phase, the voltage $V_s = V_C + V_R$. Substituting $V_R = I(t)R$ into this equation we obtain:

$$V_s = V_C + I(t)R$$

but as we have an expression for $I(t)$ in terms of V_C we get:

$$V_s = V_C + RC\frac{\partial V_C}{\partial t} \tag{3}$$

The generic solution to this differential equation is:

$$V_C = V_0 + V_b\, e^{-t/\tau_{RC}} \tag{4}$$

with $\tau_{RC} = RC$.

Discharge Phase

in the discharge phase of the capacitor, the voltage V_R across the resistor R is equal and opposite to the voltage across the capacitor, V_C as there is no other voltage source in the circuit to satisfy:

$$V_C + V_R = 0 \;\rightarrow\; V_R = -V_C \tag{5}$$

Note that the battery is disconnected in the discharge phase.

Therefore using the voltage-current relationship across a resistor we obtain:

$$I = \frac{V_R}{R} \tag{6}$$

We can now substitute Eq. 6 into Eq. 2 together with $V_C = -V_R$ to get:

$$\frac{V_R}{R} = C\frac{\partial V_C}{\partial t} \;\rightarrow\; \frac{V_C}{R} = -C\frac{\partial V_C}{\partial t} \;\rightarrow\; V_C = -\tau_{RC}\frac{\partial V_C}{\partial t} \tag{7}$$

where we define $\tau_{RC} = RC$ as the **characteristic time** of the RC-circuit.

The solution to this equation is an exponential function in time:

$$V_C = V_0\, e^{-t/\tau_{RC}} + V_b \tag{8}$$

Function of the Voltage in the Charging Phase

In the charging phase, immediately after the voltage source V_s is connected, using $t = 0$ we see that

$$I(t=0) = -\frac{CV_0}{\tau_{RC}} = -\frac{V_0}{R} \tag{9}$$

and given that there has not been any time to accumulate any charge across the capacitor, the voltage across the capacitor must be zero at $t = 0$, such that $V_C(t = 0) = 0$. Using this in Eq. 8, we obtain $V_0 + V_b = 0$. In other words our equation for the capacitor will be of the form:

$$V_C = V_s(1 - \,e^{-t/\tau_{RC}}) \tag{10}$$

As $t \rightarrow \infty$, the value of V_C will become V which would have to be V_s as there is no current flowing and the voltage drop across the resistor is zero. Given $V_s = V_C + V_R$, $V_C(t = \infty) = V_s$ and our equation for the voltage across the capacitor will be:

$$V_C = V_s(1 - \,e^{-t/\tau_{RC}}) \tag{11}$$

Function of the Voltage in the Discharging Phase

In the discharging phase, immediately after the voltage source V_s is disconnected, using $t = 0$ we see that

$$I(t=0) = -\frac{CV_0}{\tau_{RC}} = -\frac{V_0}{R} \tag{12}$$

as the capacitor was charged at V_s immediately before the voltage source was disconnected, at $t = 0$, the voltage across the capacitor has to be V_s. Using this in 8 we get:

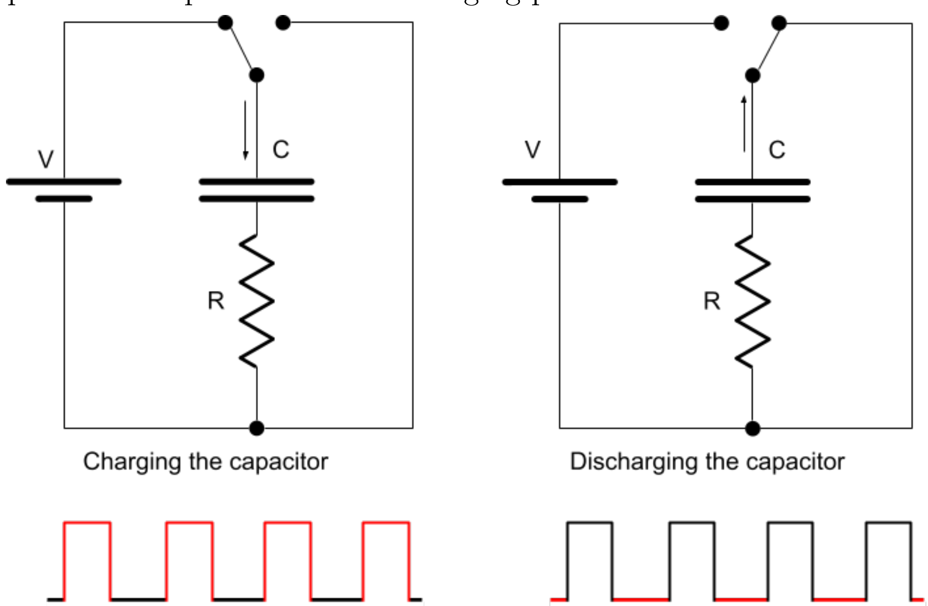
$$V_C(t=0) = V_s \;\rightarrow\; V_s = V_0 + V_b \tag{13}$$

At the same time, as $t \rightarrow \infty$, we know that the capacitor will have discharged and $V_C(t = \infty) = 0$. That means $V_0 = 0$. Therefore $V_b = V_s$ and the equation for the voltage across the capacitor is:

$$V_C(t) = V_s\, e^{-t/\tau_{RC}} \tag{14}$$

Procedure

1. Measure the resistance of the resistor using the handheld DMM.
2. Measure the capacitance of the capacitor using the capacitance measurement device.
3. Calculate τ_{RC} from these measurements.
4. Estimate the time it would take for V_C to reach 98% of V_s in the charging phase and 2% of V_s in the discharging phase using the equations above.
5. Are these times similar? Why?
6. Add these times and set the period of the function generator to this value.
7. Set the function generator to a square wave, and set the bias to 0V with the amplitude being at 5V.
8. You will use two channels of the oscilloscope. Connect the function generator output to Ch1 and connect the voltage across the capacitor to Ch2.
9. The **on** period of the square wave corresponds to the charging phase of the circuit, and the **off** period corresponds to the discharging phase as shown below



10. Observe the pattern of the V_C in the oscilloscope and ensure that the frequency of the function generator is sufficiently low that the capacitor is fully charged and discharged such that the high and low values of V_C match the function generator's settings of 0V and 5V.
11. Considering the pattern of the voltage across the capacitor in the charging phase, measure the time it takes for the capacitor to reach 63% of 5V and record this as τ_{chg}.

12. Considering the pattern of the voltage across the capacitor in the discharging phase, measure the time it takes for the capacitor to reach 37% of 5V and record this as τ_{dis}.
13. Compare τ_{dis} and τ_{chg} to τ_{RC} and discuss the percentage error.