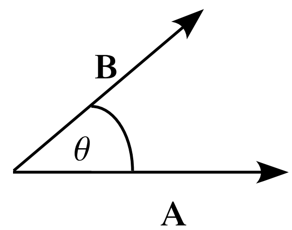
The Vector Cross Product

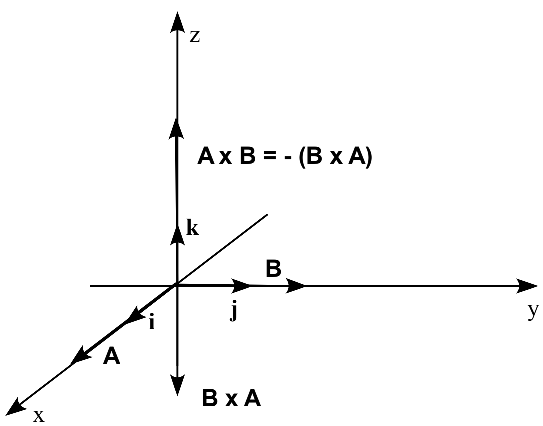
$\boxed{\vec{C} = \vec{A} \times \vec{B}}$ Vector Cross Product

$\boxed{C = AB\sin\theta}$ Magnitude of Vector Cross Product



Properties of Cross Product

1. The direction of $\vec{A} \times \vec{B}$ is given by the right-hand rule (RHR).
2. The direction of $\vec{A} \times \vec{B}$ is perpendicular to the plane formed by $\vec{A} \times \vec{B}$.
3. $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$



4. a) If $\vec{A}$ is parallel to $\vec{B}$, then $\left|\vec{A} \times \vec{B}\right| = AB\sin 0 = 0$

   b) If $\vec{A}$ is antiparallel to $\vec{B}$, then $\left|\vec{A} \times \vec{B}\right| = AB\sin \pi = 0$
5. $\vec{A}$ is perpendicular to $\vec{B}$, then $\left|\vec{A} \times \vec{B}\right| = AB\sin 90 = AB$
6. $\vec{A} \times \left(\vec{B} + \vec{C}\right) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
7. $\frac{d}{dt}\left(\vec{A} \times \vec{B}\right) = \vec{A} \times \frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt} \times \vec{B}$
8.
$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}$$
$$\hat{i} \times \hat{i} = 0, \quad \hat{j} \times \hat{j} = 0, \quad \hat{k} \times \hat{k} = 0$$

9.

$$\vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - B_xA_z) + \hat{k}(A_xB_y - B_xA_y)$$

Also,

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

<u>The Torque Vector</u>

$$\boxed{\vec{\tau} = \vec{r}\times\vec{F}}$$ Torque Vector

<u>Ex.</u>

$$\vec{r} = 3\hat{i} - \hat{k}$$

$$\vec{F} = 2\hat{i} + \hat{j} + \hat{k}$$

$$\vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ 2 & 1 & 1 \end{vmatrix} = \hat{i}\left[(0)(1) - (-1)(1)\right] - \hat{j}\left[(3)(1) - (2)(-1)\right] + \hat{k}\left[(3)(1) - (0)(2)\right]$$

$$\vec{\tau} = \vec{r}\times\vec{F} = \hat{i} - 5\hat{j} + 3\hat{k} \text{ (N}\bullet\text{m)}$$