MEASUREMENTS AND ERROR ANALYSIS LAB

OBJECTIVE

1. To learn how to use the following measuring devices and understand the uncertainties associated with them.

    a) meter stick
    b) metric ruler
    c) triple-beam balance
    d) digital balance
    e) vernier calipers
    f) micrometer

2. Use the following general error propagation equation to analyze the errors involved in making calculations involving measurements with their own uncertainty.

$$\sigma_f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial f}{\partial z}\right)^2 \sigma_z^2}$$

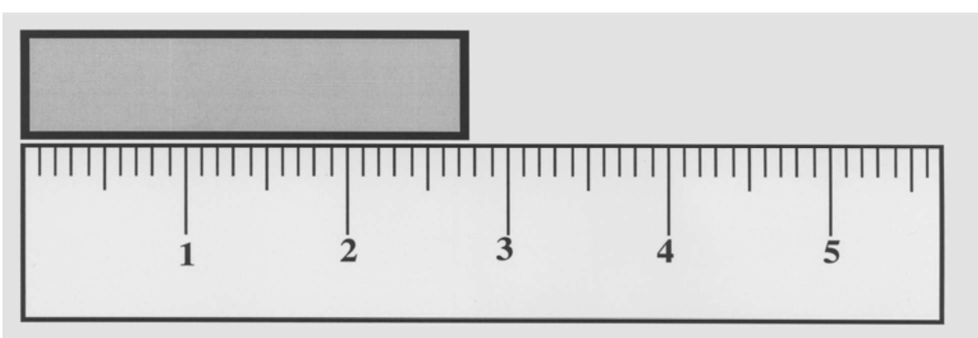
General Error Propagation Equation

THEORY

Refer to lab handout on **Error Propagation**.

<u>Using the Metric Ruler</u>

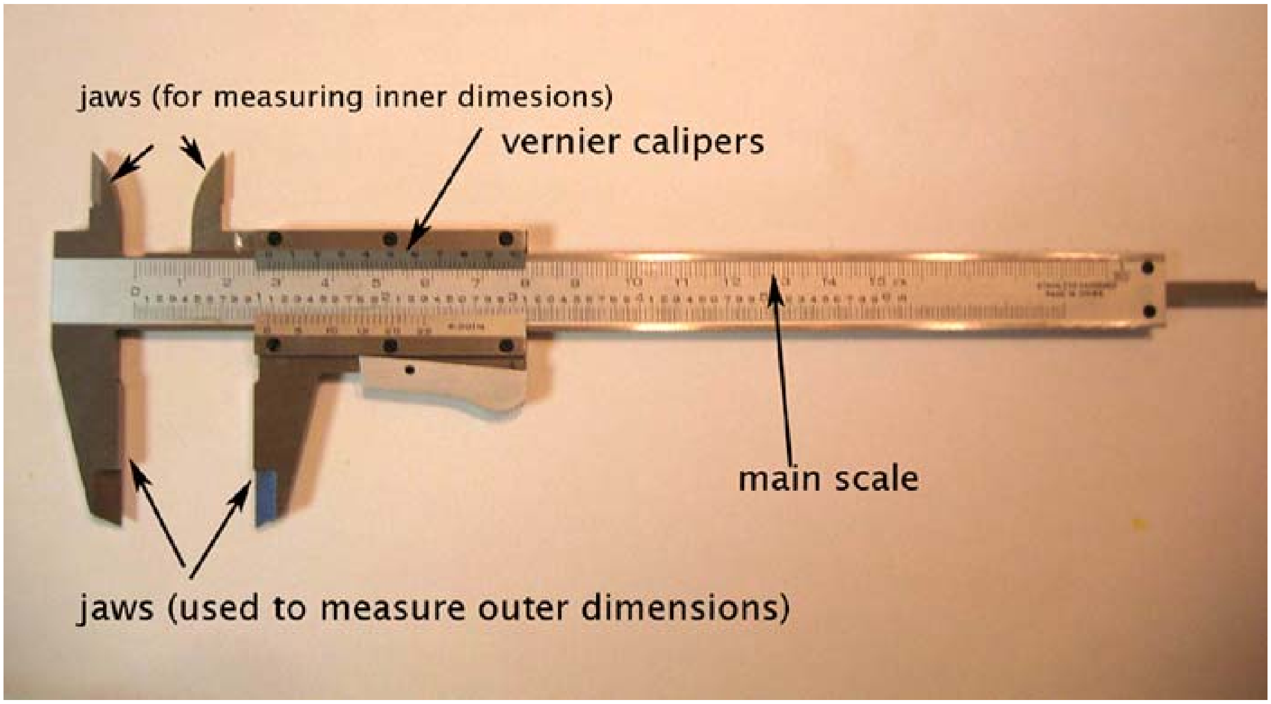
Consider the following standard metric ruler.



The ruler is incremented in units of centimeters (cm). The smallest scale division is a tenth of a centimeter or 1 mm. Therefore, the uncertainty Δx = smallest increment/2 = 1mm/2 = 0.5mm = 0.05cm. Note that a measurement made with this ruler must be stated to a tenth of a centimeter since the uncertainty is stated to a tenth of a centimeter. In the example above, the length of the object would be stated as x = 2.77 cm ± 0.05 cm.

Using the Vernier Calipers

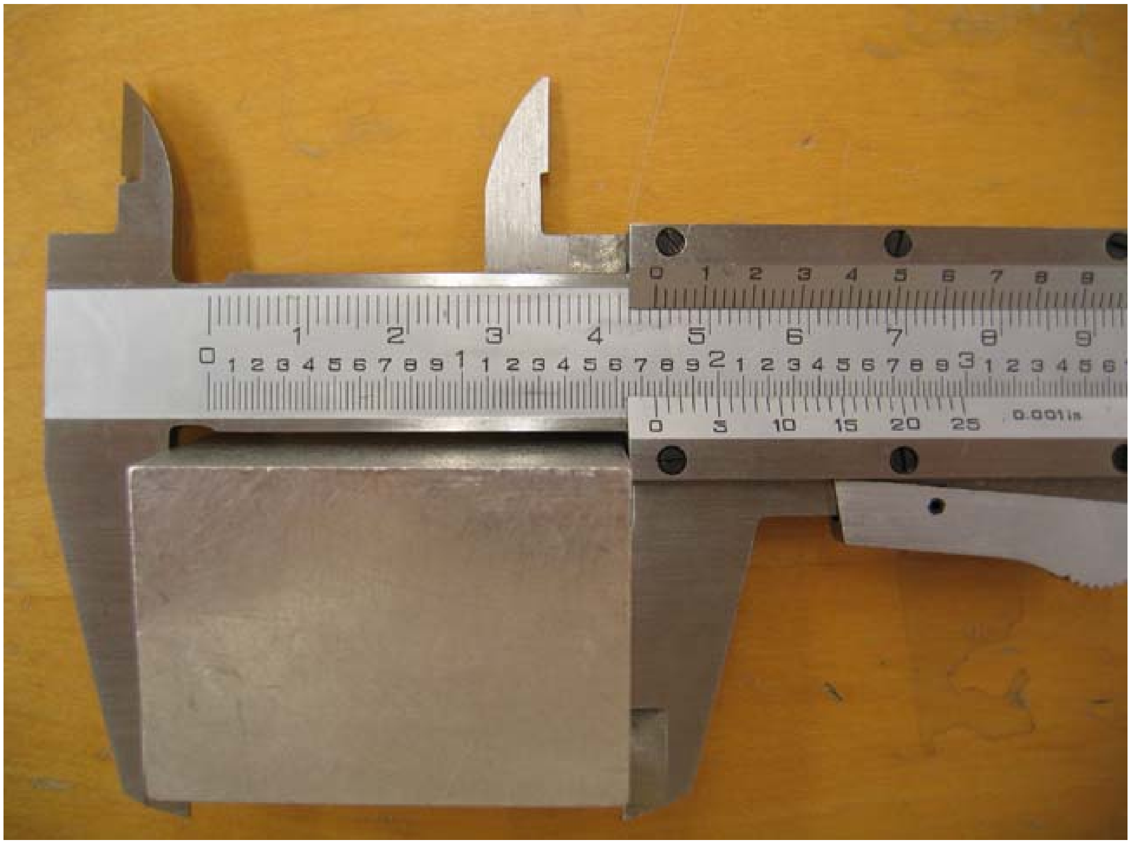
The Vernier caliper is an instrument that allows you measure lengths much more accurate than the metric ruler. The smallest increment in the vernier caliper you will be using is (1/50)mm = 0.02mm = 0.002cm. Thus, the uncertainty is $\Delta x = (1/2)0.002$ cm = 0.001 cm.



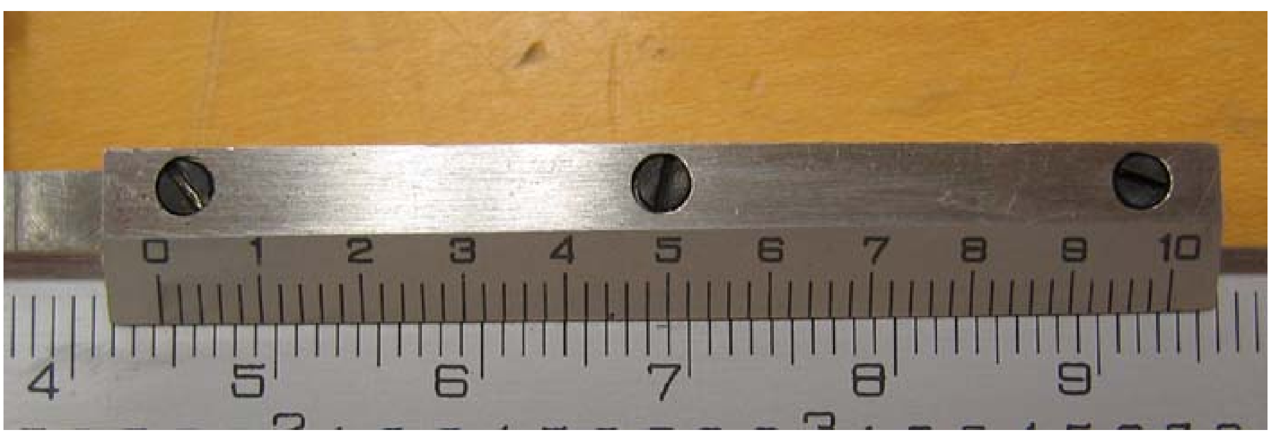
The vernier scale consists of a fixed metric scale and a sliding vernier scale. The fixed scale is divided into centimeters and millimeters, while the vernier scale is divided so that 50 divisions on it cover the same interval as 49 divisions on the main scale. Thus, the length of each scale vernier division is 49/50 the length of a main scale division. Close the jaws completely and note that the first line at the far left on the vernier scale (called the "zero" or "index" line) coincides with the zero line on the main scale. Carefully compare and see that the first vernier division is 0.02 mm short of the first main scale division, the second vernier division is 0.04 mm away from the second main scale division, and so on. If the jaws are slightly opened it is easy to tell what fraction of the main scale division the vernier index has moved by noting which vernier division best coincides with a main scale division.

A measurement is made with a vernier caliper by closing the jaws on the object to be measured and then reading the position where the zero line on the vernier falls on the main scale. The measurement is incomplete until an additional fraction of a main scale division is determined. This is obtained by noting which line on the vernier scale (0,2,4,6,8) coincides best with a line on the main scale.

As an example, let's consider measuring the length of the aluminum block below.

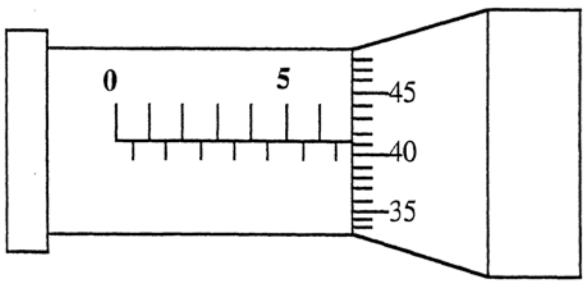


Note that the zero line on the vernier scale falls between the 4.4 cm and 4.5 cm mark on the main scale. Thus, the first significant digits are 4.4 cm. The remaining two digits are obtained by noting which line on the vernier scale (0,2,4,6,8) coincides best with a line on the main scale. Looking closely at the picture below indicates that the 46 line lines up the closest. Therefore, the reading is 4.446 cm. Or in standard form 4.446 cm ± 0.001 cm.



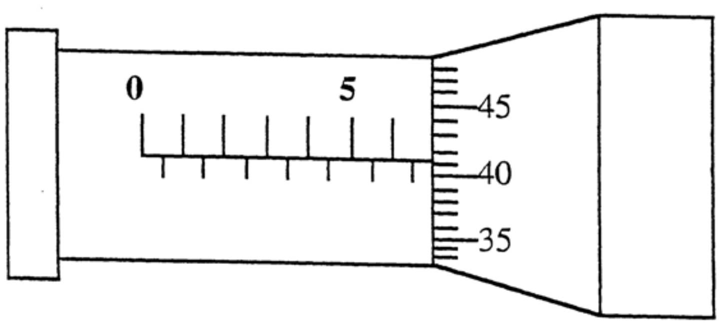
Using The Micrometer Caliper

The micrometer caliper has a linear scale engraved on its sleeve and a circular scale engraved on what is properly called the thimble. The linear scale is divided into divisions of 1 mm and is 25 mm long. Half-millimeter marks are provided below the linear scale. The circular scale has 50 divisions. One complete revolution of the circular scale moves the thimble 0.5 mm along the linear scale so that the distance between the jaws is also changed to 0.5 mm. Since the circular scale has 50 divisions, rotating it through one circular-scale division, will cause the rod to move through a distance equal to 1/50 of 0.5 mm, or 0.01 mm. Thus the numbers on the circular scale represent hundredths of a millimeter. A micrometer caliper can be used to measure lengths directly to 0.01 mm, and by estimating tenths of a circular scale division, it can be used to estimate lengths to 0.001 mm. Measurements made with a micrometer caliper can be estimated to thousandths of a millimeter. The total micrometer caliper reading is the sum of the readings on the main and circular scales.



In the figure to the left, notice the main scale which is marked with a 0 and a 5. These indicate millimeters. Below the main scale are marks at the half way points between mm marks. These l/2 mm marks are present because one revolution of the thimble moves the thimble only 1/2 mm down the barrel.

Possible settings and readings of the micrometer caliper are shown here. In the first example, the sixth main scale mark is visible just to the left of the circular scale. This means we have a reading somewhere between 6.0 and 6.5 mm. The line on the main scale points to the barrel about halfway between 19 and 20 divisions. Therefore, the micrometer calliper reading is 6.0 mm + 0.19 mm + 0.005 mm = 6.195 mm = 0.006195 meters.



In the second example, the half-millimeter mark to the right of the sixth main scale mark is visible. So the reading is somewhere between 6.5 and 7.0 mm. The line on the main scale points to a point of the barrel just slightly past the 41 mark. So we can estimate the last place of the reading to be 0.002 mm. The reading is then 6.5 mm + 0.41 mm + 0.002 mm = 6.912 mm = 0.0069 12 meters. This shows that we can estimate micrometer readings to one thousandth of a millimeter.

EQUIPMENT

1. one aluminum block
2. meter stick
3. metric ruler
4. triple-beam balance
5. digital balance
6. vernier calipers
7. micrometer

PROCEDURE

(for this lab any measurements and calculations should be stated in the standard form of: measurement = $x_{best} \pm \Delta x$)

Part I (Using Measuring Devices)

1. Learn to use all the measuring devices listed above.
2. Calculate the uncertainties of all measuring devices you will be using.

Part II (Volume of Table-Top)

1. Measure the dimensions of your table-top with the meter stick.
2. Using the error propagation equation derive an expression for the uncertainty σ_v for the volume of the table top.
3. Calculate the volume of the table top.
4. Calculate the uncertainty σ_v of the table top.

Part III (Density of Aluminum block)

1. Measure the dimensions of the aluminum block with the metric ruler, vernier calipers, and micrometers
2. Measure the mass with the block with the digital balance and triple-beam balance.
3. Calculate the volume and uncertainty of the block using the dimensions obtained from the metric ruler, vernier calipers, and micrometers. (Calculate the uncertainty σ_v of the block using the derived equation in Part II.)
4. Using the error propagation equation derive an expression for the uncertainty σ_ρ for the density of the block in terms of the uncertainty of the mass σ_M and the uncertainty of the volume σ_V.
5. Calculate the density and uncertainty of the block by using the measurements obtained from the **triple-beam balance and metric ruler**.
6. Calculate the density and uncertainty of the block by using the measurements obtained from the **digital balance and vernier caliper** .
7. Calculate the density and uncertainty of the block by using the measurements obtained from the **digital balance and micrometer**.

<u>Data Analysis</u>

1. Calculate the % error between your calculate value of density and the expected value of 2.699 g/cm^3.
2. Which of the 3 densities gave the most accurate answer? Was this what you expected why or why not? Explain!
3. Was the propagating error involved in calculating the density significant with any combination of the measuring devices? Explain.
4. What were the random errors involved and how did they affect the density and uncertainty calculation?
5. What systematic errors were involved?
6. Comment on any other sources of error that could have been involved.